Journal of Engineering Physics and Thermophysics, Vol. 72, No. 4, 1999

BEHAVIOR OF A SYSTEM OF UNIAXIAL FERROPARTICLES IN A ROTATING LIQUID MATRIX

B. É. Kashevskii

UDC 532.584:538.6

Behavior of a system of single-domain ferromagnetic particles with easy-magnetization-axis-type magnetic anisotropy in a rotating fluid matrix is considered in a transverse magnetic field that is weak compared to the effective magnetic anisotropy field of a particle. The dynamics of a separate particle and orientational state of the system are considered with regard for Brownian rotational diffusion. It is found that a small deviation from the conventional rigid dipole model leads, at frequencies exceeding a certain critical value, to appearance of two attracting stationary states of the easiest magnetization axis that are situated in the plane perpendicular to the field and approach, with the growth of a particle, one or another direction of the matrix rotation axis. It is shown that this circumstance can radically change the behavior of a system of Brownian particles and, thus, magnetic and hydrodynamic properties of the ferrosuspension.

Introduction. Stable colloidal suspensions of single-domain ferromagnetic particles (magnetic liquids) were synthesized to provide solutions to purely practical problems by using the possibilites afforded by the combination of fluidity and considerable magnetic susceptibility (whose values are four orders of magnitude higher than those of natural liquid magnetics) [1]. Further investigations of ferrosuspensions revealed a whole spectrum of basic physical, physicochemical, and hydrodynamic problems (see [2, 3]). The mutual effect of the mechanical motion of the medium and orientational motion of the particle ensemble in the magnetic field is one of these problems.

The hydrodynamic aspect of the problem consists in the fact that the effect of internal orientational degrees of freedom leads to breakdown of the symmetry condition for electromagnetic and viscous stress tensors which is basically inherent in conventional fluids and radically modifies the suspension behavior. The back-action of the flow on the orientational state of the particle ensemble is mostly investigated as an accompanying factor to hydrodynamics. At frequencies low compared to the characteristic time of establishment of orientational equilibrium it permits a rather simple and universal description [4]. Investigation of the high-frequency behavior – even in the simplest case of spherical particles with uniaxial magnetic anisotropy – meets substantial difficulties connected with the nonlinear character of the particle dynamics and the magnetic hysteresis of its crystalline matrix [5, 6].

To date, a number of high-frequency internal rotational effects in magnetic colloids, including the intriguing phenomenon of negative viscosity, are investigated [7-9]. The description of the high-frequency dependence is substantially simplified for an ideal system of particles with magnetic dipoles rigidly frozen in the crystalline matrix. The rigid dipole model is most investigated and is most frequently used in particular problems [2, 8, 10-13]. Strictly speaking, the condition of a rigidly frozen dipole is equivalent to tending to infinity the effective anisotropy field H_a of a particle which fixes the direction **e** of the magnetic moment along the direction **n** of the easiest magnetization of the particle. In practice, depending on the material, H_a has a rather moderate value of the order of $10^2 - 10^3$ Oe. The practical condition of applicability of the rigid dipole model to an actual system is given in the form of a more relaxed requirement of localization of the magnetic moment in the vicinity of one of the directions of the easiest magnetization axis [2, 3]. It is evident that the wide applications of the rigid dipole model to quasirigid systems are based on the intuitive concept that the restricted internal orientational freedom of the magnetic moment, allowing for its small deviations from the selected direction within a particle, should not substantially affect

Academic Scientific Complex "A. V. Luikov Institute of Heat and Mass Transfer" of the National Academy of Sciences of Belarus, Minsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 72, No. 4, pp. 749-756, July-August, 1999. Original article submitted July 16, 1998.

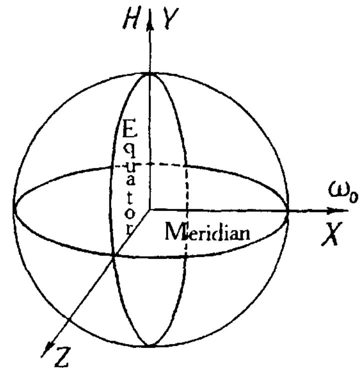

Fig. 1. Schematic of the problem.

quantitative characteristics and especially the character of the behavior of the system. The investigation carried out in the present work completely disproves this concept.

The problem of the dynamics of a rigid dipole in a rotating volume (in a vortexed flow) of a fluid under the effect of the magnetic field normal to the rotation axis (vortex) has [10] low- and high-frequency stationary solutions for orientation of the magnetic moment separated by a certain critical rotation frequency of the fluid. In the pre-critical range, the particle is at rest, and the magnetic moment lies in the equatorial plane (normal to the rotation axis, see Fig. 1) approaching with increasing frequency the meridian plane (the plane that contains the rotation axis and is normal to the field). In the post-critical stationary state, the moment is situated in the meridian plane to the left or to the right of the equator and approaches the pole (rotation axis) with increasing frequency. However, the post-critical stationary state of the rigid dipole is indifferent, and additionally, there are infinitely many types of periodic motions corresponding to the set of initial conditions. As has been pointed out in [12], the absence of magnetic moment attraction regions has fundamental importance for formation of the macroscopic state of a system of particles. An arbitrarily small Brownian motion of the particles becomes the governing factor. In the present work we show that a minimum orientational freedom of the magnetic moment in the particle body makes the high-frequency stationary states globally attractive. The decay time of their perturbations decreases with increasing field strength and can become smaller than the characteristic time of the Brownian rotational diffusion. This circumstance, as is shown by Brownian dynamics simulations, can radically change the orientational state of the ensemble of particles and, thus, high-frequency magnetic and hydrodynamic properties of the suspension.

1. Orientational Dynamics Equations for a Uniaxial Ferroparticle. We consider a spherical homogeneously magnetized grain of a ferromagnetic material having a uniaxial magnetic anisotropy with anisotropy energy density K. The grain energy in the external field $\mathbf{H} = H\mathbf{h}$

$$U = -\, mH\,(\mathbf{eh}) - Kv\,(\mathbf{en})^2$$

determines the effective orienting field ($H_a = 2Kv/m$)

$$\mathbf{H}_{\text{eff}} = -\,\partial U/\partial \mathbf{m} = H\mathbf{h} + H_a\,(\mathbf{en})\,\mathbf{n}\,.$$

Since the mechanical motion of the particle is substantially slower than the solid-state relaxation of the magnetic moment, the latter is constantly in equilibrium with the effective field. If in this case the strength of the external field is small compared to the strength of the effective field of the magnetic anisotropy of the particle ($\lambda = H/H_a < 1/2$ [14]), the direction of the magnetic moment is uniquely related to the field direction and one of the directions of the easily magnetized axis. With an accuracy up to λ^2 we find from the condition $\mathbf{e} = \mathbf{H}_{\text{eff}}/H_{\text{eff}}$ that

$$\mathbf{e} = \lambda\,[1 - \lambda\,(\mathbf{nh})]\,\mathbf{h} + \left[1 - \lambda\,(\mathbf{nh}) + \frac{3}{2}\lambda^2\left((\mathbf{nh})^2 - \frac{1}{3}\right)\right]\mathbf{n}\,. \tag{1}$$

For the electrodynamic force moment rotating the particle, it follows from (1) that

$$\mathbf{L} = \mathbf{m} \times \mathbf{H} = mH \left[1 - \lambda\,(\mathbf{nh}) + \frac{3}{2}\lambda^2 \left((\mathbf{nh})^2 - \frac{1}{3} \right) \right] [\mathbf{n} \times \mathbf{h}]\,. \tag{2}$$

Let the grain considered be suspended in a Newtonian fluid rotating at a frequency $\omega_0 \perp \mathbf{H}$. By neglecting inertia, we obtain an equation of motion for a particle by equating to zero the total force moment including the orienting moment L from (2) and the moment of viscous friction forces $-6v\eta(\omega - \omega_0)$. By using Eq. (2) and introducing the time scale $t^* = 1/\omega^*$ based on the critical synchronism frequency for a rigid dipole $\omega^* = mH/6v\eta$ [10], we write the equation of motion in the following undimensioned form ($\nu = \omega/\omega^*$ and $\nu_0 = \omega_0/\omega^*$):

$$\nu = \nu_0 + \Phi\,\mathbf{n} \times \mathbf{h}\,, \quad \Phi = 1 - \lambda\,(\mathbf{nh}) + \frac{3}{2}\lambda^2 \left((\mathbf{nh})^2 - \frac{1}{3} \right). \tag{3}$$

Equation (3) along with the kinematic relationship $d\mathbf{n}/d\tau = \nu \times \mathbf{n}$ leads to the following equation of motion for the unit vector $\mathbf{n}$ (τ is expressed in units of t^*):

$$\frac{d\mathbf{n}}{d\tau} = \nu_0 \times \mathbf{n} - \Phi\mathbf{n} \times [\mathbf{n} \times \mathbf{h}]\,. \tag{4}$$

We also write the relationship

$$\mathbf{e} \times \mathbf{h} = \Phi\mathbf{n} \times \mathbf{h}\,, \tag{5}$$

valid for every time instant. By introducing Cartesian coordinates with the X-axis directed along ν_0, Y-axis directed along $\mathbf{h}$, and Z-axis directed along $\nu_0 \times \mathbf{h}$ (see Fig. 1), we write Eq. (5) in the form of a system of equations for x, y, and z-components of the vector $\mathbf{n}$:

$$\dot{x} = -\Phi(\lambda, y)\,xy\,, \quad \dot{y} = -\nu_0 z + \Phi(\lambda, y)\,(1 - y^2)\,, \quad \dot{z} = \nu_0 y - \Phi(\lambda, y)\,zy\,. \tag{6}$$

Here the dot over the letter denotes the time derivative, and the function $\Phi(\lambda, y)$ is defined by (3) with $\mathbf{nh} = y$.

2. Stationary Orientational States. Attraction Effect Due to the Small Orientational Freedom of the Magnetic Moment. Equations (6) describe the motion of the end of the vector $\mathbf{n}$ on a sphere with unit radius. In the stationary state ($\dot{x} = \dot{y} = \dot{z} = 0$) they assume the form

$$xy = 0\,, \quad \nu_0 z - \Phi(\lambda, y)\,(1 - y^2) = 0\,, \quad y\,[\nu_0 - \Phi(\lambda, y)\,z] = 0\,. \tag{7}$$

As follows from the first of Eqs. (7), the particle axis in the stationary state can lie in either the equatorial ($x = 0$) or meridional ($y = 0$) plane. The position on the equator is characterized by the angle θ of deviation from the field direction. The deviation increases with frequency according to $\Phi(\lambda, \cos\theta)\sin\theta = \nu_0$. In the critical point where the quantity $\Phi(\lambda, \cos\theta)\sin\theta$ assumes its maximum value, $\theta_c = \pi/2 + \lambda + O(\lambda^2)$ and $\nu_c = 1$. It follows from the second of Eqs. (7) that two stationary states appear on the meridian at frequencies $\nu_0 \geq \Phi_0$ ($\Phi_0 = \Phi(\lambda, 0)$):

$$y = 0\,, \quad z = \Phi_0/\nu_0\,, \quad x = \pm\,(1 - \Phi_0^2/\nu_0^2)^{1/2}\,. \tag{8}$$

It should be pointed out that in the case of a finite degree of freezing, there is a narrow frequency range ($1 - \lambda^2/2 < \nu_0 < 1$) within which equatorial and meridional states coexist, whereas for a rigid dipole ($\lambda = 0$) they merge in the bifurcation point $\nu_0 = 1$. According to Eq. (6), the partricle axis approaches one of the two opposite rotation directions with increasing frequency. From Eq. (3) and the kinematic relationship $d\mathbf{n}/d\tau = \nu \times \mathbf{n}$ we find that in the stationary state the particle is uniformly rotated about the easiest magnetization axis at a rate

$$\boldsymbol{\nu} = \mathrm{sign}\,(\boldsymbol{\nu}_0\,\mathbf{n})\,\nu\mathbf{n}\,, \quad \nu = (\nu_0^2 - \Phi_0^2)^{1/2}\,. \tag{9}$$

Let us consider the stability of stationary states. The stability of the equatorial state is evident, and the evolution of a small perturbation ξ of the meridional state $\mathbf{n}_0$, as can be found from linearized Eqs.(7) and (8), is described by the relationship

$$\dot{\zeta} = \zeta_0 \exp\left(\frac{1}{2}\Phi_0'\tau\right)\ [\mathbf{h}\cos(\nu_p\tau + \gamma) + \mathrm{sign}\,(x)\ [\mathbf{n}_0 \times \mathbf{h}]\sin(\nu_p\tau + \gamma)] \tag{10}$$

where ζ_0 and γ are constants, $\Phi_0' = d\Phi/dy$ at $y = 0$, and $\nu_p = [\nu_0^2 - \Phi_0^2 - (\Phi_0'/2)^2]^{1/2}$. This equation describes precession of the particle axis about the position $\mathbf{n}_0$ at a frequency ν_p and with an amplitude variation increment $\Phi_0'/2 = -\lambda/2 = -H/2H_a$. For a rigid dipole ($\lambda = 0$), the state considered is neutral; however, in an actual system ($\lambda > 0$) it is attractive. Numerical experiments with equations of motion (6) make it possible to conclude that the left and right high-frequency equilibrium states on the meridian are strongly attractive for arbitrary motions of the easiest magnetization axis which start on the corresponding hemisphere.

3. Algorithm for Description of the Brownian Rotational Dynamics. The finite-difference algorithm for description of the orientational dynamics of Brownian particles is based on computation of elementary rotations of particles $\Delta\varphi^\alpha$ (α being the particle number) at the time step Δt as a sum of random orientational displacements and displacements under the effect of regular force moments [15]. By using Eq. (3) and the relationship $\Delta\varphi = \omega\Delta t$, we have

$$\Delta\boldsymbol{\varphi}^\alpha = \left(\omega_0 + \frac{mH}{6\nu\eta}\Phi\,\mathbf{n}\times\mathbf{h}\right)\Delta t + \delta\boldsymbol{\varphi}^\alpha . \tag{11}$$

Random rotations $\delta\varphi_i^\alpha$ about the i-th Cartesian axis are chosen from a Gaussian distribution with a zero mean and variance

$$\langle\delta\varphi_i^2\rangle = 2D_r\Delta t , \tag{12}$$

where $D_r = kT/6\nu\eta$ is the rotational diffusion coefficient. The change in the position of the unit vector of the easiest magnetization axis as a result of a small rotation is determined by the kinematic relationship $\Delta\mathbf{n} = \Delta\varphi \times \mathbf{n}$. By using the above time scale t^*, we write the finite-difference model of the orientational dynamics in the form ($\xi = mH/kT$)

$$\Delta\mathbf{n}^\alpha = [\boldsymbol{\nu}_0 \times \mathbf{n}^\alpha + \Phi\,(\mathbf{n}^\alpha\mathbf{h})\,(\mathbf{h} - \mathbf{n}^\alpha\,(\mathbf{n}^\alpha\mathbf{h}))]\,\Delta t + \delta\boldsymbol{\varphi}^\alpha \times \mathbf{n}^\alpha , \tag{13}$$

$$\delta\varphi_i^\alpha = \sqrt{2\xi^{-1}\Delta\tau}\ R_i^\alpha ,\ \langle R_i^2\rangle = 1 , \tag{14a}$$

For an ensemble of N particles, N normally distributed numbers R_i are obtained from $N + 1$ numbers c_i normally distributed on the interval [0, 1] according to the relationshhip $R_i = \sqrt{-2\ln c_i}\,\sin 2\pi c_{i+1}$. Magnetization of the system (in units of saturated magnetization nm) is calculated by averaging relationship (1) over the ensemble of particles

$$\langle e_i\rangle = \langle n_i\rangle + \lambda\,[h_i - \langle n_in_j\rangle h_j] - \lambda^2\left[\langle n_j\rangle h_ih_j + \frac{1}{2}(1 - 3\,\langle n_in_jn_k\rangle h_jh_k)\right] . \tag{15}$$

To test the algorithm, we calculated the equilibrium magnetization that for any λ should vary with the field strength according to Langevin's law

$$\langle\mathbf{e}\rangle = \mathbf{h}L(\xi) ,\ L(\xi) = \mathrm{ctanh}\,(\xi) - 1/\xi . \tag{16}$$

The effect of the number of particles in the ensemble on the result of computations virtually vanishes after N = 500. The direct use of algorithm (14a)-(15) provides convergence to the analytical result (16) only in the range of rather strong fields ($\xi > 5$). Convergence in weak fields can be provided by dividing the Brownian motion on the step $\Delta\tau$ into several (r) successive independent displacements, which improves statistical properties of sampling. Instead of (14a) we use

$$\delta\varphi_i^\alpha = \sqrt{\left(\frac{2\Delta\tau}{\xi r}\right)}\ \sum_{k=1}^{r} (R_i^\alpha)_k , \tag{14b}$$

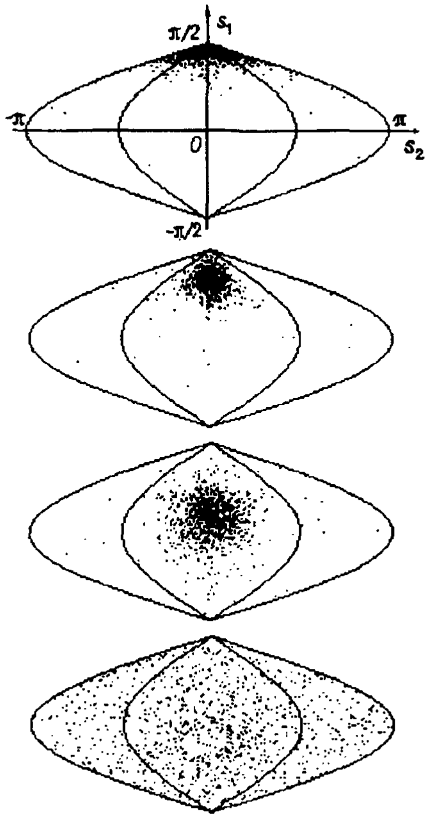

Fig. 2. Effect of the matrix rotation frequency ν_0 on the dynamically equilibrium distribution of ends of unit vectors of the easiest magnetization axes of ferroparticles on the surface of a unit sphere in the field $\xi = 20$ in the case of rigidly (left panels) and partially frozen (right panels) magnetic moments in the particle body. $\nu_0 = 0$, 0.5, 1, and 5 (from top to bottom).

where $(R_i^\alpha)_k$ is the k-th sample of the normal distribution of random numbers with $\langle R_i^2 \rangle = 1$. With increasing r, divergence between analytical and numerical magnetization values decreases and, in addition, convergence is provided with decreasing step length. The dependence on the step length virtually disappears when the maximum of the values of $\Delta\tau$ and $2\Delta\tau/\xi$ becomes smaller than 0.005. For $N = 500$, $\Delta\tau = 0.001$, and $\xi = 0.2$, calculated magnetization values exceed Langevin's values by 110, 30, 15, and 5% for $r = 1$, 11, 21, and 25, respectively. Coincidence with the analytical results within the limits of 5% for $\xi = 0.2$, 1, 5, and 10 is achieved at $r = 25$, 11, 3, and 1, respectively (for $\xi = 10$, the deviation < 1%).

4. Character of Particle Distribution in the Orientational Space. To provide a pictorial presentation of the character of the particle distribution in the orientational space, we present results of the numerical simulation in the form of a distribution of points presenting positions of easiest magnetization axes on a unit sphere. On the spherical surface, we introduce the coordinates s_1 and s_2 of the mapping point by placing the origin at the point of intersection of the equator and meridian and by denoting by s_1 the path length along the equator to the point of its intersection with the plane $Y = y$ and by s_2 the path length from this point to the mapping point along the intersection of the plane $Y = y$ with the sphere. A uniform distribution of mapping points corresponds to a uniform orientational distribution of the easiest magnetization axes on the (s_1, s_2)-plane. Snapshots of easiest magnetization axes ($N = 1000$) in the state of dynamic equilibrium are shown in Fig. 2 for $\lambda = 0$ (left panels) and $\lambda = 0.4$ (right panels) for $\nu_0 = 0$, 0.5, 1, and 5, and $\xi = 20$. The external curve in Fig. 2 bounds the entire surface of the sphere, whereas the internal curve bounds the surface of the hemisphere $z > 0$ (see Fig. 1). As is evident, for the rigid dipole model ($\lambda = 0$), the distribution is close to axisymmetric and broadens with increasing frequency. A deviation in the behavior of nonrigid dipoles is observed already at the frequency $\nu_0 = 1$ (the distribution is stretched along the meridian) and it increases with increasing frequency. At the frequency $\nu_0 = 5$, instead of the virtually isotropic

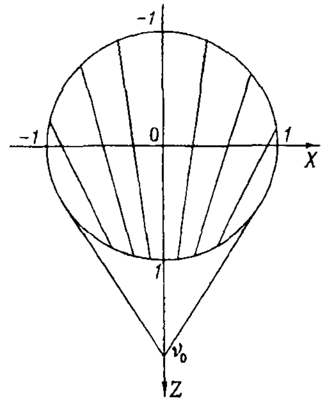

Fig. 3. Family of trajectories of motion of the end of the unit vector of the magnetic moment of the rigid dipole projected on the meridian plane upon rotation of the matrix with the frequency ν_0.

distribution observed for rigid dipoles, we have a pronounced biaxial distribution whose characteristic directions are close to the fluid rotation axis, which corresponds to stationary states (8) for $\nu_0 = 5$. The reason for the existence of a preferential direction for rigid dipoles at post-critical frequencies is not so straightforward, but it can be easily understood from an analysis of the dynamic solution for a single particle. According to [10], at frequencies $\nu_0 > 1$, the rigid dipole has an infinite number of periodic motions. For each of them, the particle rotation axis lies in the meridian plane and makes the angle φ in the range of $-\varphi_0$ to $+\varphi_0$ with the fluid rotation axis, and it should be noted that the bounds of the above range correspond to stationary states of the easiest magnetization axis and are determined, according to (8), by the equation $\sin \varphi_0 = 1/\nu_0$. The end of a unit vector of the easiest magnetization axis describes a circle, and all planes of motion intersect along the line $Z = \nu_0$, $Y = 0$ (see Fig. 3). The points of tangency correspond to stationary states. As is evident, the trajectories become denser toward the meridian plane the stronger and the closer the frequency is to unity. We find the stationary distribution of particles over trajectories $\rho(\varphi)$ in the following manner. We assume that the original orientational distribution is uniform and that there are no directed particle flows from one trajectory to another during the motion. We also take into account that the angle between the plane $X = 0$ and the plane of a given trajectory equals $-\varphi$. Then the fraction of particles $\rho(\varphi)d(\varphi)$ in the interval of trajectories of φ to $\varphi + d\varphi$ equals the ratio of the sphere surface area between the corresponding planes to the entire sphere area. From this we find

$$\rho(\varphi) = \frac{1}{2}\nu_0 \cos(\varphi), \quad |\sin(\varphi)| \le \frac{1}{\nu_0}. \tag{17}$$

The condensation of trajectories toward the center can be characterized by the relationship $\rho(0)/\rho(\varphi_0) = \nu_0/(\nu_0^2 - 1)^{1/2}$. Upon approaching the bifurcation point ($\nu_0 = 1$), this ratio tends to infinity, and in the limit $\nu_0 \to \infty$ it tends to unity (trajectories are uniformly distributed over the sphere). In addition, the distribution of particles along the trajectory φ is also nonuniform. This is connected with the nonuniform character of motion due to the decelerating or accelerating effect of the field on portions $z > 0$ and $z < 0$, respectively. By introducing the angle θ_1 of deviation of the variable component of the vector **n** from the vertical direction upon moving along the trajectory φ, we, based on (6), obtain the equation for it:

$$\dot{\theta}_1 = \nu_0 \cos\varphi - (1 - \nu_0^2 \sin^2\varphi)^{1/2} \sin\theta_1 .$$

The probability $\rho_1(\theta_1)d\theta_1$ of observing a particle with the angle in the range of θ_1 to $\theta_1 + d\theta_1$ equals the ratio of the time of its presence in this range to the motion period, which is identical for all trajectories and equals $\Theta = 2\pi(\nu_0^2 - 1)^{-1/2}$ [10]. Therefore,

$$\rho_1(\theta_1) = \frac{1}{\Theta\dot{\theta}_1} = \frac{(\nu_0^2 - 1)^{1/2}}{2\pi\,(\nu_0 \cos\varphi - (1 - \nu_0^2 \sin^2\varphi)^{1/2} \sin\theta_1)}. \tag{18}$$

The maximum of the quantity $\rho(\theta_1)$ is achieved on a meridian, and it decreases with increasing frequency. By combining results (17) and (18), we conclude that at frequencies $\nu_0 > 1$, rigid dipoles have a preferential orientation along the positive direction of the Z-axis (i.e., along $\nu_0 \times \mathbf{h}$). The orientation anisotropy increases upon approaching the bifurcation point and broadens with increasing frequency, which agrees with the results of the numerical simulation presented in Fig. 2. The attracting effect of stationary states is characterized by the decay time of their perturbations (see Section 2), which can be presented in the form $\tau_s = \tau_B(8\sigma/\xi^2)$, where $\tau_B = 3v\eta/kT$ is the characteristic time of Brownian rotational diffusion of the particle, and $\sigma = Kv/kT$ is the ratio of the particle anisotropy energy to the energy of the thermal motion. Since the thermal motion prevents orientation of particles, the attraction intensity can be characterized by the ratio $\gamma = \tau_B/\tau_s = \xi^2/8\sigma$. By using $\lambda = \xi/2\sigma$, we also write $\gamma = \lambda^2\sigma/$ or $\gamma = \lambda\xi/4$. Since the limiting allowed value of λ equals 0.5, and the limiting field strength $\xi = \sigma$, the limiting value of the attraction parameter $\gamma_{max} = \sigma/8$. For the above-considered system ($\xi = 20, \lambda = 0.4$), the attraction parameter $\gamma = 2$, and the particle anisotropy parameter $\sigma = 25$. According to these concepts, a decrease in λ and increase in ξ by a similar factor should retain the distribution of the easiest magnetization axes. Indeed, calculations for $\lambda = 0.2$, $\xi = 40$ and $\lambda = 0.1$, $\xi = 80$ yielded distributions visually identical to that presented in Fig. 2 for $\lambda = 0.4$, $\xi = 20$. It should be noted that the time necessary for establishment of stationary distribution (expressed in units of the inverse critical synchronism frequency t^*) increases as $\propto 1/\lambda$ with decreasing λ (at γ = const) (as does the above-calculated perturbation decay time in the stationary state of a "cold" particle). In the case considered ($\gamma = 2$) it approximately equals $\xi/2$.

Thus, a small orientational freedom of the magnetic moment in the particle body can have important macroscopic consequences. To date, in particular, the phenomenon of the major slowing down of the magnetic relaxation rate of a ferrocolloid premagnetized to saturation along its rotation axis has been observed.

NOTATION

m and v, magnetic moment and volume of particle; ω and ν, angular and undimensioned angular velocities of particle; n, numerical concentration of particles; η, viscosity; k, the Boltzmann constant; T, temperature. Indices: a, anisotropy; B, Brownian; c, critical; s, solid-state; eff, effective; p, precession; r, rotation.

REFERENCES

1. J. L. Neuringer and R. E. Rosensweig, *Phys. Fluids*, 7, 1927-1937 (1964).
2. M. I. Shliomis, *Uspekhi Fiz. Nauk*, **112**, 427-458 (1974).
3. E. Ya. Blum, M. M. Maiorov, and A. O. Tsebers, *Magnetic Fluids* [in Russian], Riga (1989).
4. B. M. Berkovsky, A. N. Vislovitch, and B. E. Kashevsky, *IEEE Trans. Magnetics*, **MAG-16**, 329-342 (1980).
5. A. V. Petrikevitch and Yu. L. Raikher, *J. Magn. Magn. Mat.*, **32**, 79-81 (1983).
6. B. E. Kashevskii, *Magn. Gidrodin.*, No. 2, 52-60 (1986).
7. B. E. Kashevskii, *Dokl. Akad. Nauk SSSR*, **264**, 574-577 (1982).
8. J.-C. Bacri, R. Perzynski, M. J. Shliomis, and G. I. Burde, *Phys. Rev. Lett.*, 75, 2128-2131 (1995).
9. R. E. Rosensweig, *Science*, **271**, 614-615 (1996).
10. W. F. Hall and S. N. Busenberg, *J. Chem. Phys.*, **51**, 137-145 (1969).
11. M. A. Martsenyuk, Yu. L. Raikher, and M. I. Shliomis, *Zh. Eksp. Teor. Fiz.*, **65**, 834-841 (1973).
12. T. P. Lyubimova, D. V. Lyubimov, and M. I. Shliomis, Magnetodynamics of Ferrosuspensions in a Rotating Field [in Russian], *Preprint*, Institute of the Mechanics of Continuous Media of the Ural Branch of the Academy of Sciences of the USSR, Sverdlovsk (1985).
13. H. Brenner and M. H. Weissman, *J. Coll. Interf. Sci.*, **41**, 499-531 (1972).
14. L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* [in Russian], Moscow (1982).
15. E. Dickinson, *J. Chem. Soc., Faraday Trans.*, **81**, 2-11 (1985).